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Thirteenth Lecture 24-2-1947

A Gravitational Field is Regarded as an Affine Connection

$\overline{ds}^2 = \overline{dt}^2 - \overline{d\sigma}^2 \qquad \overline{d\sigma}^2 = \overline{dx_1}^2 + \overline{dx_2}^2 + \overline{dx_3}^2$

$dt = dx_4$: ds is invariant.

ds may be +ve, −ve or zero

ds is experimentally present in the special theory of relativity: The analogue of $\overline{dx_1}^2 + \overline{dx_2}^2 + dx_3^2$ in general curvilinear coordinates is a general quadratic form $g_{ik}\, dx_i\, dx_k$.

This idea of a "Riemanian metric" must be put in harmony with the idea of affine connection: $\boxed{24.2.1947}$

a grav^l field is regarded as an affine connec^n. The path of the particles & the particles themselves are 4 dim^l geodesics: No grav^l field corresponding to str. lines; & corresponds to integrable symmetric affinity;

	(a) Newtontian way	(b) Analytically we have
(I) No field :	ϕ = Cons:	
(II) Field but no matter (empty space)	$\nabla^2 \phi \beta = 0$	

→ we shd contemplate what is known as a metric:

1) symetric affinity $\Gamma^i_{kl} = \Gamma^i_{lk}$

2): $B^i_{klm} = 0$

1) Symetry affinity $\Gamma^i_{kl} = \Gamma^i_{lk}$

2) Not the R.C. Tensor vanishes but its contraction: $R_{kl} = B^m_{klm} = 0$.

This is less than $B^i_{klm} = 0$

16 eqns $R_{kl} = 0$ are not sufficient & hence

2

	Newtonian Method	General
(III) General	$\nabla^2 \phi = 4\pi\rho$	$R_{kl} = \alpha T_{kl}$

These are only first tentative suggestions & we still have to correct them.

Along every geodesic of every affine connecⁿ there is a metric given by the distance bet. two neighbouring pts C & D of one & the same geodesic. There is no possibility whatsoever of comparing elements of length along two different geodesics.

Integrating this invariant ds along any geodesic we find the length of this curve. Denoting $dx_4 = dt$

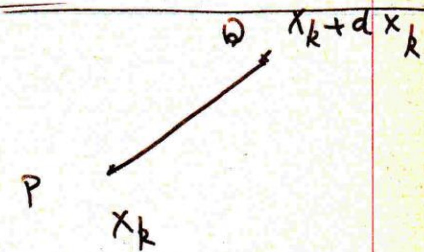
$\frac{dx_1}{dx_4}$, $\frac{dx_2}{dx_4}$, $\frac{dx_3}{dx_4}$ is the vel. in 3 dimensional space but is not a tensor in 4 dim.

We can form: $\frac{dx_1}{ds}, \frac{dx_2}{ds}, \frac{dx_3}{ds}, \frac{dx_4}{ds}$. & this is a contr. vector (4 compts). This is introduced from 2 pts of view. The 2nd suggestion of this metric is the special or restricted theory of relativity.

Two tasks: 1, How this invariant comes from the special ~~[illegible]~~ theory of relativity 2, In what way can the

Relativity

- together with $T^{i}_{\ kl} = T^{i}_{\ lk}$



Remark: The general theory of relativity may be regarded as a restriction of the special theory of relativity "which applies to the whole flat space time" on to an infinitismial region of curved space time so small that variations of the gravitional field of force ~~during~~ through that region may be neglected: In this sense the terminology: "restricted" & "general" is rather misleading.

idea of affine connection & this metric can be adapted with each other ? & without contradictions : ?

(I) How this invariant arises from the special theory of relativity ?

(a) Michelson & Morley experiment.
(b) Binary stars
(c) Aberration of light. | In the special theory of relativity

We have 2 systems, an inertial sys. of coor one w.r. to which the ordinary Newtonian laws of mechanics holds : at rest or in uniform motion w.r. to

$x_1 \quad x_2 \quad x_3 \;\&\; (x_4 = t)$

We measure the time & the coordinates s.t. the velocity of light is 1 : In the rest^d theory of relativity we contemplate linear transformations with cons. coeff^ts which are interpretted as going over to another inertial system of coord's moving with a certain velocity relative to the first system.

$$\left.\begin{aligned} x_1' &= x_1 - u_1 x_4 \\ x_2' &= x_2 - u_2 x_4 \\ x_3' &= x_3 - u_3 x_4 \\ x_4' &= x_4 \end{aligned}\right\}$$

This is the Newtonian law of transform^n.

a light cm is a unit of time

$= \frac{1}{3\times10^{10}}$ of a sec.

a light year is a unit of length

a light second „ also a unit of length

The Lorentz transformation differs from this in that x_4' depends on x_4 together with ~~the~~ adding other terms (dep. on v)

The great service of the res. theory of rel. is the increase of mass with velocity. There is a group of experiments agreeing with this fact. "Bocher's experiments on fastly moving electrons".

Since these linear transf. include a transf^n of the time there is a (1,1) corresp. bet. pts as described in the 2 systems. we mean space time pts. In partic^n it follows that the distance bet. 2 events $A = (0,0,0,0)$ & $B(x_1, x_2, x_3, x_4)$. The dis. in space bet^n A & B will be given by, $x_1^2+x_2^2+x_3^2$

$$x_1^2+x_2^2+x_3^2 \neq x_1'^2+x_2'+x_3'^2$$

$$x_4^2 \neq x_4'^2$$

but

$$x_1^2+x_2^2+x_3^2-x_4^2 = x_1'^2+x_2'^2+x_3'^2-x_4'^2$$

$$-x_1^2-x_2^2-x_3^2+x_4^2 = -x_1'^2-x_2'^2-x_3'^2+x_4'^2$$

This is a characteristic of a L.T. It describes a L.T. exhaustively. The math.l description is analogous to what is known as "a rotation of 3 dimension space":

$$X'_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3$$
$$X'_2 =$$
$$X'_3 = a_{31}x_1 + \cdot\ \cdot \quad + a_{33}x_3$$

is a linear transformation for which $X^2+Y^2+Z^2$ is invariant. It can be shown tha this is an exhaustive description.

In general $\mathrm{Det}\ a_{ik} = \pm 1$ We restrict ourselves to "$\mathrm{Det}\ a_{ik} = +1$". This can be reached gradually by infinitisimial trans.s Those corresponding to $\mathrm{Det}\ a_{ik} = -1$ Can't be obtained in this way: . Those in which $\mathrm{Det}\ a_{ik} = +1$ form a "Group".

$$X_1^2 + X_2^2 + X_3^2 - X_4^2 = \text{invariant}.$$

$X_1^2 + X_2^2 + X_3^2 + X_4^2 = \text{Const}$ is a rotation in 4 dimensions.

$$X_1' = a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + a_{14}X_4$$
$$X_2' =$$
$$X_3' =$$
$$X_4' = a_{41}X_1 + a_{42}X_2 + a_{43}X_3 + a_{44}X_4$$

Let us write it in the form

$x_1' = a_{11}x_1 + a_{12}x_2 + \cdots \qquad - i\,a_{14}(ix_4)$

$ix_4' = i a_{41}x_1 + i a_{42}x_2 + \cdots\cdots + a_{44}(ix_4)$

$x_1^2 + x_2^2 + x_3^2 + (ix_4)^2 =$ invariant.

The determ^{t} of the transformation $= +1$

a_{44} must be positive so that the time does not run in 2 diff. directions in both systems: From this inf^{n} we can write the L.T. used in the proof of various formula.

'Rotation in even dimensional space has no fixed axis ex: 2 dimensions

'Rotation in odd dimensional space has a fixed axis. In rotating a globe round its centre to any other position is done with 2 pts on the surface of the globe kept fixed.

$x_1 \quad x_2 \quad x_3 \quad x_4$

~~rot~~ a L.T. can be considered as a rot^{n} in the x_2, x_3 plane & a rotation in the x_1, x_4 plane: This has the characteristics of a L.T.

$x_1' = x_1\cos\phi + x_4\sin\phi$	$x_1' = x_1\cosh\phi - x_4\sinh\phi$
$x_1' = -x_1\sin\phi + x_4\cos\phi$	$x_2' = -x_1\sinh\phi + x_4\cosh\phi$
$x_1'^2 + x_4'^2 = x_1^2 + x_4^2$	$x_1'^2 - x_4'^2 = x_1^2 - x_4^2$
$\cos^2\phi + \sin^2\phi = 1$	$\cosh^2\phi - \sinh^2\phi = 1$