Twelfth Lecture

Physical Interpretation of Affine Connection

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Relation bet. Geodesics & Path of Particles:

This arises from the fact that a particle under no forces moves in 3 dimensions in a straight line & with a uniform velocity.

Gravitation can be described geometrically as a field of affine connection. The geodesics are the tracks of particles. If there is no field at all then according to Newtonian mech.s a particle moves in a st. line with uniform vel.

. All particles move on the same tracks indep.^{t} of the nature of particles.

In the particular case of no field the affinity is symmetric & integrable.

What are the laws to which the affine fields must be subjected so as to get the ordinary laws of gravitation. One thing is that the Newtonian Case follows as a 1st approx^{n}

Consider a particle m with potential $-\frac{m}{r}$

(a) in the case of no field the potential ϕ = Constant.

(b) In the Case of field of outside the masses $\nabla^2 \phi = 0$

(c) In the case of field inside the masses $\nabla^2 \phi = 4 \pi K \rho$

K the gravit^{n} Cons., ρ the density b is a special case of c when $\rho = 0$

What are the corresponding eqns for Γ^i_{kl}

for the case (a) we have the integrable symetric affinity: A frame should exist in which the geodesics are str. line. $\underbrace{B^i_{klm} = 0}$

$\underbrace{\Gamma^i_{kl} = \Gamma^i_{lk}}_{24\ \text{eqns}}$. These are 80 eqns

These are the necessary & sufft conditions for integrability.

(a) is a special case of b.

Huge number of possible generalisations exist

(1) One of the generalisns is to drop the 24 eqns $\Gamma^i_{kl} = \Gamma^i_{lk}$ but it did not work. it was tried by Einstein in 1928

(2) $B^i_{klm} B^k_{pqr} = 0$

$$B^i_{kli} = R_{kl} = 0$$

$$R_{kl} + B^i_{klm} B^k_{iqr}$$

$$B^i_{klm} = -\frac{\partial \Gamma^i_{kl}}{\partial x_m} + \frac{\partial \Gamma^i_{km}}{\partial x_l} + \Gamma^\alpha_{km} \Gamma^i_{\alpha l} - \Gamma^i_{\alpha m} \Gamma^\alpha_{kl}$$

$$R_{kl} = B^i_{kli} = -\frac{\partial \Gamma^\alpha_{kl}}{\partial x_\alpha} + \frac{\partial \Gamma^\alpha_{k\alpha}}{\partial x_l} + \Gamma^\alpha_{k\beta} \Gamma^\beta_{\alpha l} - \Gamma^\beta_{\alpha\beta} \Gamma^\alpha_{kl}$$

. It is Linear in the derivs & quadratic in the Γ's

$$S_{lm} = -\frac{\partial \Gamma^{\alpha}{}_{\alpha l}}{\partial x_m} + \frac{\partial \Gamma^{\alpha}{}_{\alpha m}}{\partial x_l}$$ Answer

Let us try the possibilities of putting

$R_{kl} = B^{i}{}_{kli} = 0$ $\Gamma^{i}{}_{kl} =$ and/or $\Gamma^{i}{}_{lk}$ $S_{kl} = 0$

Consider R_{kl} in the case of symetric affinity

The first ~~& third~~ & fourth terms are symetric in k & l but not the second terms: let us split R_{kl} into its symetric & skew parts. Using the hook & underlining for skew & sym^c proper^s

$$R_{kl} = R_{\check{kl}} + R_{\underline{kl}}$$ refused

$$R_{\check{kl}} = \frac{1}{2}\left(\frac{\partial \Gamma^{\alpha}{}_{k\alpha}}{\partial x_l} - \frac{\partial \Gamma^{\alpha}{}_{l\alpha}}{\partial x_k}\right) = -\frac{1}{2}S_{kl}$$

Let us take away $S_{kl} = 0$ since with the symetry condition it is entialed.

attractive features of this assumption is ~~that~~ that the vanishing of R_{kl} is the thing we require for "energy"!

"Conservation of C.G during radiation processes!" Radiation has a pressure.

See Born Appendix ... page...

$$B^{i}{}_{klm} = \Gamma^{i}{}_{km,l} - \Gamma^{i}{}_{kl,m} + \Gamma^{\alpha}{}_{km}\Gamma^{i}{}_{\alpha l} - \Gamma^{i}{}_{\alpha m}\Gamma^{\alpha}{}_{kl}$$

(i) Linear in the derivatives

(ii) Quadratic in the affinity components

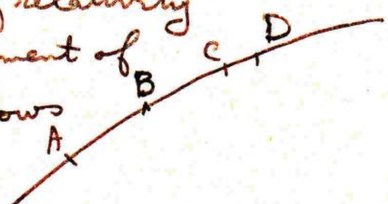
Kinetic energy can be zero in one system & not zero in another system: Energy can be the time Component of a vector of which the first 3 Comp^ts are mom^m Components:

$$R_{kl} = \alpha T_{kl}$$

T_{kl} being the energy mom^m tensor

Admitting this system of 40 eq^ns we want besid them something more. 2 circumstances combine & lead us to another geometrical conception "The Riemanien metric". One is of a theoritical nature. the other is connected with the restricted theory of relativity

Along a geodesic an element of length ds which allows us to compare lengths along the same geodesic.



"It is better to say that the geodesics are the particles rather than the geodesics are paths of particles

Instead of using the disgusting expressions $\frac{dx_1}{dx_4} . \frac{dx_2}{dx_4} . \frac{dx_3}{dx_4} . \frac{dx_4}{dx_4}$ Can now be replaced by $\frac{dx_1}{ds} . \frac{dx_2}{ds} . \frac{dx_3}{ds} . \frac{dx_4}{ds}$ which are the Components of velocity & they have simple laws of transform^n when compared with the previous quantities

Remark:

The affine Connection defines system of geodesics: The distance bet. 2 neighbouring points of the geodesic is given by $ds^2 = g_{ik}\, dx^i\, dx^k$. Thus an affine Connection defines a metric only along the geodesics. This may be called "the trace of a geodesic" : ~~Its~~ It does not give us any information at all about the distance between two neighbouring pts ~~to~~ not on the same geodesic:

$$\frac{dx^\alpha}{ds} = \left(\frac{v^r}{c\sqrt{1-\frac{v^2}{c^2}}}, \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$R_{kl} - \frac{1}{2} R\, g_{kl} + \lambda\, g_{kl} = \kappa T_{kl}$$

$$\boxed{(T_{kl})_{;l} = 0}$$

$$\nabla^2 \psi = 4\pi k \rho$$

$$(T_{kl})_{,l} = 0$$