The Deformation Structure of the Even-Even p- and s-d Shell Nuclei

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Abstract
In order to investigate the deformation structure of the even-even p- and s-d shell nuclei: ^{8}Be, ^{12}C, ^{20}Ne, ^{24}Mg ^{28}Si, ^{32}S and ^{36}Ar, we have calculated two characteristics for these nuclei by using different models that depend on the shape of the nucleus. In the case where the nucleus is assumed to be deformed and has an axis of symmetry, we have applied the concept of the single-particle Schrödinger fluid for the calculation of the nuclear moment of inertia together with the usual method of calculating the electric quadrupole moment in this case. In the case where the nucleus is assumed to be deformed and has not an axis of symmetry, the nuclear superfluidity model and the cranked Nilsson model are applied for the calculations of the moments of inertia and the quadrupole moments of these nuclei, respectively. The analysis of the obtained results showed that some of these nuclei may have axes of symmetry.

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1. Introduction
In the shell model, there is a core made up of paired nucleons. This core may be spherically symmetric in which case it gives rise to the spherically symmetric of the independent particle model or axially symmetric, as in the Nilsson model [1] which also be referred to as the deformed independent particle model.

Magic nuclei are spherical at equilibrium. Nuclei with only a few particles outside closed shells have also spherical shapes in their ground states. The lowest 2^+ states in the even-even nuclei are related to the quadrupole vibrations of the nuclear surface; they represent the degrees of freedom, which are easiest to excite. The features just described characterize the vibrational nuclei with only a few particles or holes in unified shells and with a spherical equilibrium form.

The spherical nuclear shape becomes less and less stable when the number of particles or holes in the unfilled shells is increased. The outside nucleons interact by the residual interaction. The interaction results in a correlated motion of the particles which, in turn, leads to a nuclear non sphericity. The likelihood of a stable deformed nuclear shape is a fast increasing function of the number of particles in unfilled shells. Consequently, nuclei with many neutrons and protons in unfilled shells have non spherical, ellipsoidal shapes. The first 2^+ states of such even-even nuclei have very small energies; the sequence of the 2^+, 4^+, 6^+, etc., levels can be interpreted as a rotational band corresponding to the rotation of the whole nucleus [2]. Nuclei with these properties are called rotational nuclei.

Many of the light nuclei are spherical. This is due to the success of the shell model, which is based on states in a field of spherical symmetry. According to the basic ideas of quantum mechanics the concept of rotation in a spherically symmetric system is meaningless. However, in an elongated nucleus the concept of rotation is meaningful, and the nucleus can rotate about an axis perpendicular to the axis of symmetry.

The analysis of nuclear spectra within the framework of the shell model is feasible only for relatively few nuclei: those which are fairly close to magic numbers. The shell model has a limited range of useful applicability, covering nuclei in the vicinity of closed shells only. The complexity of nuclear spectra increases very rapidly as we go farther and farther away from closed shell [2].

However, as we go farther away from closed shells some new, very simple and systematic, features start to show up for some nuclei [2]. This is true for nuclei with mass number A in the range $155 < A < 185$, for $A > 225$, for nuclei in the s-d shell $19 \leq A \leq 25$, and for p shell nuclei $9 \leq A \leq 14$. Odd-A nuclei in these regions are characterized by exceptionally large positive quadrupole moments, doubly even nuclei in the same region all have a rather low-lying first excited state with $J = 2^+$. Its essential point is the assumption that we are dealing here with nuclei with a permanent prolate deformation. In the Bohr-Mottelson model the nucleus, which resembles a short thick cigar, is assumed strongly enhanced. A description of these nuclei and many other deformed nuclei has been given by a model proposed and developed by a nucleus to rotate slowly around an axis perpendicular to its symmetry axis and there is a very strong correlation among all the nucleons in the nucleus, which manifests itself in a relatively slow rotation of the deformed overall pattern into which the nucleons shape their combined mass distribution.

The basic ideas concerning non spherical nuclei have been most completely described by A. Bohr [2]. A non spherical nucleus is characterized by the moment of inertia about the axis perpendicular to the symmetry axis of the nucleus, its magnetic dipole moment and its electric quadrupole moment. The elongation of the nucleus is related to the interaction between the surface and the nucleons outside closed shells.

It is well known that the nucleons inside the nucleus occupy approximately 1/50 of the volume of the nucleus. It is not surprising to find that nucleon properties are maintained inside the nucleus. In particular this situation is responsible for the fact that the magnetic dipole moments of nucleons inside nuclei are the same as for free nucleons. In accordance with the above, we describe the motion of each nucleon individually in a common field. Because the surface of deformed nuclei is distorted at some moment, the potential felt by the nucleons is not spherically symmetric. The next step was to include the effect of deformation on the single particle motion.

Hartree-Fock calculations revealed that ground states of alpha-like nuclei in the p-shell are axially deformed [3]. One of the most successful models for generating realistic intrinsic single particle states of deformed nuclei is that first proposed by Nilsson [1]. This model was limited to nuclei with axially symmetric quadrupole deformations, where the deformation is measured by the deformation parameter β. Positive values of the deformation parameter correspond to prolate deformation and negative values to oblate deformation.

In treating the internal motion in the nucleus, it is assumed that the individual nucleons move independently in a certain fixed non-spherical field of the nucleus. The Hamiltonian of the internal motion can then be represented, as in the ordinary model, in the form of a sum of one-particle Hamiltonians. According to Nilsson's model the nucleons inside the nucleus are moving independently in an averaging field in the form of anisotropic oscillator, with $\omega_x = \omega_y \neq \omega_z$, added to it a spin-orbit term and a term proportional to the square of the orbital angular momentum of the nucleon. The nucleon energy eigenvalues and eigenfunctions are then obtained by solving the time-independent Schrödinger wave equation in spherical polar coordinates and applying the method of diagonalizing the matrices [4].

The large quadrupole moments observed in some nuclei, which do not belong to closed shells, implied a collective deformation and thereby a rotational degree of freedom. The most central parameter of collective rotation is the quadrupole moment and the moment of inertia of deformed nuclei [2,5]. The study of the velocity fields for the rotational motion of the axially symmetric deformed nuclei led to the formulation of the so-called Schrödinger fluid [6,7]. Since the Schrödinger-fluid theory is an independent particle model, the cranking model approximation for the velocity fields and the moments of inertia play the dominant role in this theory.

In the case of deformed nuclei, the theoretical question to be settled first is whether or not the nucleus has an axis of symmetry. Historically, several applications of the theory were made [8] on the assumption that the deformed nucleus does have such an axis of symmetry. Most of the work in the heavier nuclei is made on this assumption. In a field with axial symmetry, only the component of the angular momentum along the axis of symmetry is conserved. Historically, much after the application of the axially symmetric-rotor model, a systematic attempt was made by several authors, especially Davydov and his collaborators [8] to check the consequences of the general rotor Hamiltonian that has no axis of symmetry (usually called an asymmetric rotor). The total motion of the nucleons is thus composed of two parts: an internal motion with respect to the body-fixed reference frame, described by an internal wave function, and the motion of the body-fixed reference frame itself. The nuclear superfluidity model [9] and the cranked Nilsson model [10] provide us with powerful methods for calculating the nuclear moment of inertia, the electric quadrupole moment and other characteristics of deformed nuclei, having no axes of symmetry.

In this paper we have investigated the deformation structure of the even-even p- and s-d shell nuclei; namely the nuclei ^{8}Be, ^{12}C, ^{20}Ne, ^{24}Mg , ^{28}Si, ^{32}S and ^{36}Ar. Accordingly, we have calculated two characteristics for these nuclei by using different models that depend on the shape of the nucleus. In the case where the nucleus is assumed to be deformed and has an axis of symmetry, we have applied the concept of the single-particle Schrödinger fluid for the calculation of the moment of inertia. For the calculations of the electric quadrupole moment in this case we apply the usual method [1].

In the case where the nucleus is assumed to be deformed and has not an axis of symmetry, we have considered a single-particle deformed potential consisting of an anisotropic oscillator potential added to it a spin-orbit term and a term proportional to the square of the orbital-angular momentum of the nucleon to calculate the single-particle energy eigenvalues and eigenfunctions for a nucleon in a deformed non axial nucleus. As a consequence, the ground-state of the nuclei ^{8}Be, ^{12}C, ^{20}Ne, ^{24}Mg ^{28}Si, ^{32}S and ^{36}Ar are constructed and their moments of inertia are then calculated by applying the superfluidity nuclear model of Belyaev [9] as functions of the deformation parameter β, the non-axiality parameter γ and the non deformed oscillator parameter $\hbar\omega_0^0$, which is given in terms of the mass number A, the number of neutrons N and the number of protons Z [11]. Furthermore, the single-particle wave functions which are obtained in this case are used to calculate the electric quadrupole moment of the mentioned nuclei.

Moreover, we have applied the cranked Nilsson model for the calculation of the electric quadrupole moment of the mentioned seven nuclei, assuming that these nuclei are deformed and have not axes of symmetry. Accordingly, we have calculated each characteristic of the deformed nucleus by applying two methods, one of which is based on the assumption that the nucleus has an axis of symmetry and the other is based on the assumption that the nucleus has not axes of symmetry.

The calculated values of the moment of inertia and the electric quadrupole moment of the nuclei ^{8}Be, ^{20}Ne, and ^{28}Si in the case where it has been assumed that these nuclei are deformed and have axes of symmetry are in better agreement with the corresponding experimental values rather than the other two cases, a result which shows that these nuclei may be assumed that they are deformed and have axes of symmetry. On the other hand, the obtained results for the nuclei ^{12}C and ^{24}Mg showed that these two nuclei have no axes of symmetry.

2. Calculations Based on the Assumption that the Nucleus is Deformed and Has an Axis of Symmetry

2.1 The Single-Particle Schrödinger Fluid

The detailed formulation of the concept of the single-particle Schrödinger fluid from the time dependent Schrödinger equation, by suitably chosen single-particle wave function, is given by Kane and Griffin [6]. The method of applying this concept to the calculations of the nuclear moments of inertia is given by Doma [7]. The following expressions for the cranking-model and the rigid body-model moments of inertia can be easily obtained on the basis of the concept of the single-particle Schrödinger fluid [6]

$$\Im_{cr} = \frac{E}{w_0^2}\left(\frac{1}{6+2\sigma}\right)\left(\frac{1+\sigma}{1-\sigma}\right)^{\frac{1}{3}}\left[\sigma^2(1+q)+\frac{1}{\sigma}(1-q)\right]. \tag{2.1}$$

$$\Im_{rig} = \frac{E}{w_0^2}\left(\frac{1}{6+2\sigma}\right)\left(\frac{1+\sigma}{1-\sigma}\right)^{\frac{1}{3}}[(1+q)+\sigma(1-q)], \tag{2.2}$$

where q is the anisotropy of the configuration, which is defined by

$$q = \frac{\sum_{occ}(n_y+1)}{\sum_{occ}(n_z+1)}, \tag{2. 3}$$

and E is the total energy

$$E = \sum_{occ}\left[\hbar\omega_x\left(n_x + n_y + 1\right) + \hbar\omega_z(n_z + 1)\right]. \tag{2. 4}$$

In equations (2.3) and (2.4) n_x, n_y and n_z are the state quantum numbers of the oscillator. The summations in (2.3) and (2.4) are carried over all the occupied single-particle states. The method of filling these states is illustrated in [7]. Also, in (2.1) and (2.2) σ is a measure of the deformation of the potential and is defined by

$$\sigma = \frac{\omega_y - \omega_z}{\omega_y + \omega_z}. \tag{2. 5}$$

The frequencies ω_x, ω_y and ω_z are given by [1]

$$\omega_x^2 = \omega_y^2 = \omega_0^2(\delta)\left(1 + \frac{2}{3}\delta\right), \tag{2.6}$$

$$\omega_z^2 = \omega_0^2(\delta)\left(1 - \frac{4}{3}\delta\right), \tag{2.7}$$

$$\omega_0(\delta) = \omega_0^0\left(1 - \frac{4}{3}\delta^2 - \frac{16}{27}\delta^3\right)^{-\frac{1}{6}}. \tag{2.8}$$

For the non deformed frequency ω_0^0 we use the one which is given in terms of the mass number A, the number of neutrons N and the number of protons Z by [11]

$$\hbar\omega_0^0 = 38.6\, A^{-\frac{1}{3}} - 127.0\, A^{-\frac{4}{3}} + 14.75\, A^{-\frac{4}{3}}(N - Z). \tag{2.9}$$

The well-known deformation parameter β is related to the parameter δ by the following relation [1]

$$\beta = \frac{2}{3}\sqrt{\frac{4\pi}{5}}\,\delta. \tag{2.10}$$

We note that the cranking-model and the rigid body-model moments of inertia are equal only when the harmonic oscillator is at the equilibrium deformation.

2.2 The Electric Quadrupole Moment

Assuming a charge distribution in accordance with the Thomas-Fermi statistical model applied to the oscillator potential one obtains, for the case of the axially symmetric nuclei, the intrinsic quadrupole moment, to the second-order in the deformation parameter δ [1]

$$Q_0 = 0.8ZeR^2\delta\left(1 + \frac{2\delta}{3}\right), \tag{2.11}$$

where Z is the number of protons and R is to be taken equal to the radius of charge of the nucleus. The relation between the measured quadrupole moment, denoted by Q_S, and Q_0 is given by

$$Q_S = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)}Q_0, \tag{2.12}$$

where I is the spin-quantum number of the specified nuclear state and K is its component along the body-fixed z-axis. Calculating the charge radius of the nucleus, the measured quadrupole moment for a nucleus with an axis of symmetry is then obtained as function of the deformation parameter δ.

3. Calculations Based on the Assumption that the Nucleus is Deformed and Has Not an Axis of Symmetry

In the case where the nucleus is assumed to be deformed and has not, in principle, an axis of symmetry we apply two different models: the nuclear superfluidity model and the cranked Nilsson model.

3.1 The Single-Particle Potential and the Nuclear Superfluidity Model

Consider a nucleon which is moving in a deformed nuclear field whose Hamiltonian operator is given by [4]

$$H = -\frac{\hbar^2}{2m}\nabla^2 + \frac{m}{2}\omega_0^2 r^2 + C\boldsymbol{l}.\boldsymbol{s} + Dl^2 - m\omega_0^2 r^2 \beta cos\gamma Y_{2,0}(\theta,\varphi) - \frac{\sqrt{2}}{2} m\omega_0^2 r^2 \beta sin\gamma \{Y_{2,2}(\theta,\varphi) + Y_{2,-2}(\theta,\varphi)\}, \quad (3.1)$$

where $Y_{l,\Lambda}(\theta,\varphi)$ are the spherical harmonic functions, β is the deformation parameter and γ is the non-axiality parameter. The constants C and D in equation (3.1) are given by [1]

$$C = -2\chi\hbar\omega_0^0, D = -\mu\chi\hbar\omega_0^0, \quad (3.2)$$

where χ takes values in the interval $0.05 \le \chi \le 0.08$ and μ depends on the number of quanta of excitation N as given by Nilsson [1].

The Schrödinger equation representing the motion of a single nucleon in the non-axially deformed nuclear field whose Hamiltonian operator is given by equation (3.1) can be solved by applying both of the variation method, for the fifth term in equation (3.1) with respect to the eigenfunctions of the first four terms, the functions $|Nl\Lambda\Sigma\rangle$, and then the stationary non-degenerate perturbation method, for the last term in (3.1) with respect to the eigenfunctions which result from the application of the variational method, the functions $|N\Omega^\pi\rangle$. As a result, the single-particle energy eigenvalues and eigenfunctions, $|\Omega^\pi\rangle$, of a nucleon in a deformed nuclear field can be calculated for every level, with given value of the z-component of the total angular momentum Ω and parity π as functions of the potential parameters χ, and μ, the deformation parameter β, and the non-axiality parameter γ.

The moment of inertia of a deformed nucleus which has no axis of symmetry is then given by applying the nuclear superfluidity model [9], and as a result we obtain

$$\mathfrak{I}_{s.f.} = \hbar^2 \sum_{i,k} \frac{\langle i|J_x|k\rangle^2}{E_i+E_k}\left\{1 - \frac{(\zeta_i-\lambda)(\zeta_k-\lambda)+\Delta^2}{E_iE_k}\right\}, \quad (3.3)$$

where ζ_i are the eigenvalues of the self-consistent field, the eigenvalues of the Hamiltonian operator (3.1), λ is the chemical potential and the energy of elementary excitations of the nucleus, E_i, is given by

$$E_i = \sqrt{(\zeta_i - \lambda)^2 + \Delta^2}, \tag{3.4}$$

with Δ being the energy gap. The summation in equation (3.4) is taken over all states of the self-consistent field. The chemical potential λ is given by [5]

$$\sum_i \left\{1 - \frac{\zeta_i - \lambda}{\sqrt{(\zeta_i - \lambda)^2 + \Delta^2}}\right\} = \mathrm{N}_{p,n}, \tag{3.5}$$

where the summation, here, runs over all distinct neutron (or proton) energies and $\mathrm{N}_{\mathrm{p,n}}$ is the number of protons or neutrons inside the nucleus.

For the non-axial case the intrinsic quadrupole moment of a nucleus consisting of Z protons is given by [2]

$$Q_0 = \sum_{i=1}^{Z} Q_i, \tag{3.6}$$

where the single-particle operator Q_i is given by

$$Q_i = e\sqrt{\frac{16\pi}{5}} \int \left(\Psi_{\Omega^\pi}^i\right)^2 r_i^2 Y_{2,0}(\theta_i, \phi_i)\, d\tau. \tag{3.7}$$

Carrying out the integration in equation (3.7) with respect to the wave functions $|\Omega^\pi\rangle$ which is evaluated in terms of the functions $|Nl\Lambda\Sigma\rangle$, the eigenfunctions of the first four terms of the Hamiltonian (3.1), one then obtains

$$Q_i = e\sqrt{\frac{16\pi}{5}} \sum_{\alpha,\beta} C_\alpha^i C_\beta^i \langle N_\alpha l_\alpha | r^2 | N_\beta l_\beta \rangle \langle l_\alpha \Lambda_\alpha | Y_{2,0} | l_\beta \Lambda_\beta \rangle. \tag{3.8}$$

Filling the single-particle wave functions $|\Omega^\pi\rangle$ for the given nucleus in its ground-state it is then possible to calculate the quadrupole moment by calculating the necessary matrix elements of equation (3.8) and evaluating the expansion coefficients of the functions $|\Omega^\pi\rangle$ in terms of the functions $|Nl\Lambda\Sigma\rangle$ as obtained from the variational and the perturbation methods.

3.2 The Cranked Nilsson Model

Within the framework of the collective model the nucleons are assumed to move coherently. They are paired and form the nuclear core. Rotation is one degree of freedom of collective motion. The nucleus can be treated as a classical rigid body (rotor) [2]. The rotational spectrum of a pure collective rotation is given by [2]

$$E_{rot}(I) = \frac{\hbar^2}{2\mathfrak{I}} [I(I+1)], \tag{3.9}$$

where $\mathfrak{I}$ is the moment of inertia and I is the total angular momentum. The effects of rotation on the single-particle states are described within the framework of the cranking model. The behavior of nucleons is studied by rotating the nucleus with rotational frequency, ω, i.e. to crank the nucleus, hence the name cranking model. The first mathematical formulation of the model was made by Inglis [12,13] and has been further developed by Bengtsson and Frauendorf [14,15,16]. The model provides a microscopic description of the influence of rotation on single-particle motion. The rotation is treated classically and the nucleons are considered as independent particles moving in an average rotating potential. The basic developments leading to the modified single-particle oscillator potential are described in [17,18,19], while cranking was introduced in [20,21]. The single-particle Hamiltonian used here is in the form [22]

$$H^{\omega} = H^{0} - \omega j_x = H_{ho}(\varepsilon,\gamma) + 2\hbar\omega_0\rho^2\sqrt{\frac{4\pi}{9}}\,\varepsilon_4 V_4(\gamma) + V' - \omega j_x\,, \qquad (3.10)$$

where $H_{ho}(\varepsilon,\gamma)$ is the anisotropic harmonic oscillator Hamiltonian

$$H_{ho}(\varepsilon,\gamma) = \frac{p^2}{2m} + \frac{1}{2}m\{\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2\}, \qquad (3.11)$$

with the frequencies ω_x, ω_y and ω_z expressed in the quadrupole deformation parameters in the usual way with signs chosen according to the Lund convention [21,23,24]

$$\omega_j = \omega_0(\varepsilon,\gamma)\left[1 - \frac{2}{3}\varepsilon\cos\left(\gamma + \frac{2\pi v_j}{3}\right)\right], \quad j \in \{x,y,z\} \qquad (3.12)$$

with $v_x = 1$, $v_y = -1$ and $v_z = 0$. The calculations are carried out in the stretched coordinate system [17,23], $\xi = x\sqrt{M\omega_x/\hbar}$ etc., and the higher multipoles in the potential are also defined in these coordinates; i.e., the spherical harmonics $Y_{\lambda\mu}$ are functions of the angles θ_t and φ_t where the index t refers to the stretched system. The corresponding coordinate radius is denoted by ρ in (3.10).

The hexadecapole potential is defined to obtain a smooth variation [25] in the γ-plane so that the axial symmetry is not broken for $\gamma = -120^o, -60^o, 0^o$ and 60^o. It is of the form [26,27].

$$V_4 = a_{40}Y_{4,0} + a_{42}\left(Y_{4,2} + Y_{4,-2}\right) + a_{44}\left(Y_{4,4} + Y_{4,-4}\right)\,, \qquad (3.13)$$

where the a_{4i} parameters are chosen as

$$a_{40} = \frac{1}{6}(5\cos^2\gamma + 1), \qquad a_{42} = -\frac{1}{12}\sqrt{30}\sin 2\gamma, \qquad a_{44} = \frac{1}{12}\sqrt{70}\,sin^2\gamma. \qquad (3.14)$$

The term V', which is also defined in the stretched coordinates, is introduced to reproduce the level ordering as observed in nuclei,

$$V' = -\kappa(N)\hbar\omega_0^o\{2\boldsymbol{\ell}_t.\boldsymbol{s} + \mu(N)(\ell_t^2 - \langle\ell_t^2\rangle_N)\}. \qquad (3.15)$$

The parameters κ and μ might either be given the same values for each shell or, alternatively, as indicated in (3.15), they can be made dependent on the main oscillator quantum number $N = N_t$.

The diagonalization of the Hamiltonian (3.10) gives the eigenvalues E_i^{ω} and the eigenvectors χ_i^{ω}. Furthermore, the single-particle energies in the laboratory system and the single-particle spin contributions m_i are obtained as

$$E_i = \langle \chi_i^{\omega} | H^0 | \chi_i^{\omega} \rangle , \quad m_i = \langle \chi_i^{\omega} | j_x | \chi_i^{\omega} \rangle, \tag{3.16}$$

where H^0 is the static single-particle Hamiltonian.

The radius, expressed as a function of the angles in the stretched-coordinate system, is obtained by requiring a constant value for the potential in (3.10) (neglecting the $V' - \omega j_x$ term):

$$\rho^2 \propto \frac{1}{1-\frac{2}{3}\varepsilon\sqrt{\frac{4\pi}{5}}\left(\cos\gamma Y_{2,0}-\frac{1}{\sqrt{2}}\sin\gamma\left(Y_{2,2}+Y_{2,-2}\right)\right)+\varepsilon_4\sqrt{\frac{4\pi}{9}}V_4(\gamma)} , \tag{3.17}$$

where the spherical harmonics are functions of the angles θ_t and φ_t and the harmonic-oscillator part of the potential is expressed in ε and γ. From the definition of the stretched coordinates, it is straightforward to express the angles θ_t and φ_t in the corresponding angles in the spherical system, θ and φ.

Because of the incompressibility of nuclear matter, the nuclear volume is kept constant when the nucleus is deformed. This is achieved by varying the frequency $\omega_0(\varepsilon, \gamma, \varepsilon_4)$ from its value for a spherical shape, ω_0^0. The integration of the nuclear volume is most easily performed in the stretched-coordinate system and then multiplied with the corresponding Jacobian, a constant that is proportional to $\sqrt{\omega_x \omega_y \omega_z / \omega_0^3}$.

From the single-particle wave functions the electric (or mass) quadrupole moment may be calculated as

$$Q_2 = \sum_{occ} p_i \langle \chi_i^{\omega} | q_2 | \chi_i^{\omega} \rangle \tag{3.18}$$

where $p_i = 1$ for protons and 0 (or 1) for neutrons.

4. Results and Conclusions

In Table-1 we present the calculated values of the reciprocal moments of inertia of the nuclei ^{8}Be, ^{12}C, ^{20}Ne, ^{24}Mg , ^{28}Si, ^{32}S and ^{36}Ar by using the concept of the single-particle Schrödinger fluid for both of the cranking-and the rigid-body models. The values of the deformation parameter β and the oscillator parameter $\hbar\omega_0^0$ are also given in Table-1. In Table-1 we also give the reciprocal values of the moments of inertia of the mentioned nuclei by using the nuclear superfluidity model together with the corresponding experimental values. The values of the potential parameter χ, the deformation parameter β and the non axiality parameter γ are also given in Table-1.

According to previous works [4,7] the parameters χ, μ and β are allowed to take on the values $\chi = 0.05, 0.06, 0.07, \text{and } 0.08,\ \mu = 0,$ for $N = 0,1 \text{ and } 2;$ and $\mu = 0.35$ for $N = 3$, (where N is the shell-number of quanta of excitation) and β takes values in the interval $-0.50 \leq \beta \leq 0.50$ with a step 0.01. The corresponding values of the potential parameter χ and the deformation parameter β, and the non deformed oscillator parameter $\hbar\omega_o^o$ for each nucleus are also given in Table-1. It is seen from Table-1 that the values of the deformation parameter β for the two nuclei ^{12}C and ^{28}Si are negative while these values are positive for the five nuclei ^{8}Be, ^{20}Ne, ^{24}Mg, ^{32}S and ^{36}Ar, a result which is in agreement with the Hartree-Fock calculations for the four nuclei ^{12}C, ^{28}Si, ^{8}Be and ^{20}Ne, which have shown also that the four nuclei are axially symmetric [3].

Furthermore, it is seen from Table-1 that the calculated values of the cranking-model reciprocal moments of inertia are in better agreement with the corresponding experimental values rather than the other values. The disagreement between the values of the rigid-body reciprocal moment of inertia and the corresponding experimental values is due to the fact that the pairing correlation is not taken in concern in this model [28].

In Table-2 we present the calculated values of the electric quadrupole moment of the nuclei ^{8}Be, ^{12}C, ^{20}Ne, ^{24}Mg , ^{28}Si, ^{32}S and ^{36}Ar according to formulas (2.11) and (2.12) for the axially symmetric case .

In Table-3 we present the calculated values of the electric quadrupole moment of the nuclei ^{8}Be, ^{12}C, ^{20}Ne, ^{24}Mg , ^{28}Si, ^{32}S and ^{36}Ar for the non axial case, by using the cranked Nilsson model.

According to the obtained results of the two calculated deformation characteristics, the assumption that the nuclei ^{8}Be, ^{20}Ne, ^{28}Si, ^{32}S and ^{36}Ar are deformed and have axes of symmetry seems to be the more reliable assumption, a result which has been also obtained for some nuclei in the s-d shell [29]. On the other hand, the two nuclei ^{12}C and ^{24}Mg may have not axes of symmetry.

Table-1 Moments of inertia of ^{8}Mg, ^{12}C, ^{20}Ne, ^{24}Mg , ^{28}Si, ^{32}S and ^{36}Ar, by using the single-particle Schrödinger fluid and the nuclear superfluidity model

Nucleus	^{8}Be	^{12}C	^{20}Ne	^{24}Mg	^{28}Si	^{32}S	^{36}Ar
$\hbar\omega_0^0$ (MeV)	11.36	12.24	11.88	11.55	11.22	10.91	10.62
β	0.277	- 0.434	0.436	0.331	-0.233	0.487	0.324
$\frac{\hbar^2}{2\Im_{cr}}$ (KeV)	446.5	742.4	279.85	233.8	324.67	371.3	374.1
$\frac{\hbar^2}{2\Im_{rig}}$ (KeV)	282.6	212.3	172.32	69.86	57.97	41.96	152.3
$\frac{\hbar^2}{2\Im_{S.F.}}$ (KeV)	428.71	750.0	293.87	237.72	329.90	388.32	371.08
χ	0.06	0.08	0.06	0.08	0.06	0.06	0.06
β	0.16	-0.24	0.41	0.21	-0.2	0.32	0.22
γ	10^0	30^0	10^0	30^0	10^0	15^0	10^0
$\frac{\hbar^2}{2\Im_{exp}}$ (KeV)	446.19	750.0	279.90	237.90	324.60	371	374

Table-2 Electric quadrupole moment of ^{8}Mg, ^{12}C, ^{20}Ne, ^{24}Mg , ^{28}Si, ^{32}S and ^{36}Ar for the axially-symmetric case

Nucleus	β	R_{RMS}	Q_0	Q_S	Q_{exp} barns
^{8}Be	0.277	3.00	0.08	-0.022	N/A
^{12}C	-0.42	3.32	-0.21	0.08	0.06
^{20}Ne	0.69	3.94	0.81	-0.231	-0.23
^{24}Mg	0.37	4.18	0.59	-0.179	-0.166
^{28}Si	-0.28	4.4	-0.58	0.164	0.165
^{32}S	0.178	4.92	0.52	-0.149	-0.149
^{36}Ar	-0.11	5.12	-0.39	0.11	0.11

Table-3 The electric quadruple moments by using the cranked Nilsson model

nucleus	γ	β	Q_{cal}	Q_{exp} barns
^{8}Be	10^0	0.277	-0.048	N/A
^{12}C	30^0	- 0.434	0.0595	0.06
^{20}Ne	-10^0	0.333	-0.2295	-0.23
^{24}Mg	30^0	0.331	-0.1638	-0.166
^{28}Si	5^0	-0.233	0.1653	0.165
^{32}S	10^0	0.45	-0.1492	-0.149
^{36}Ar	10^0	-0.38	-0.109	0.11

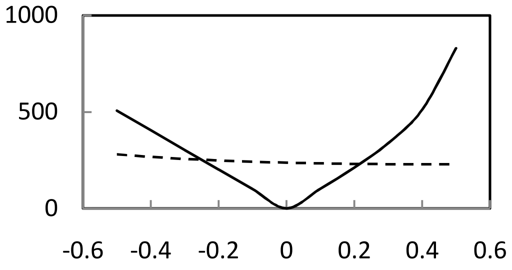

Fig. 1 The nucleus $^{8}_{4}Be$.

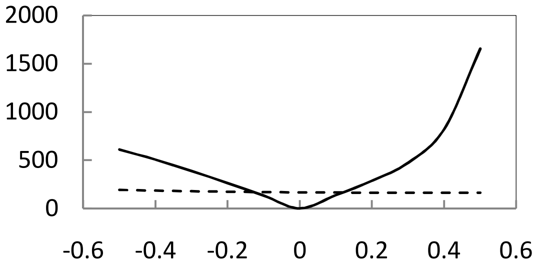

Fig. 2 The nucleus $^{12}_{6}C$.

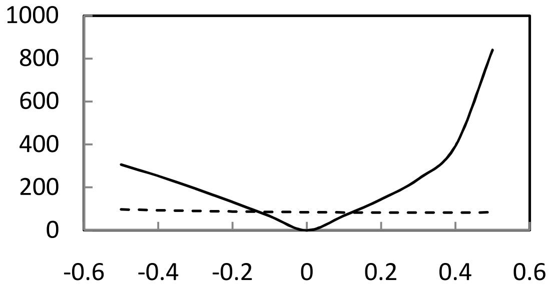

Fig. 3 The nucleus ${}^{20}_{10}Ne$.

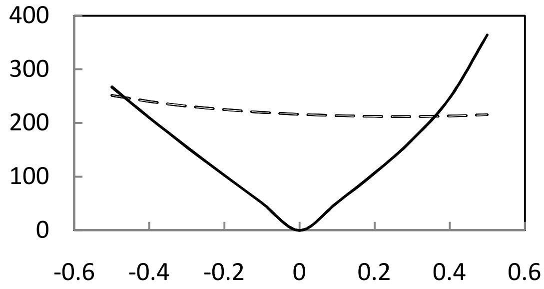

Fig. 4 The nucleus ${}^{24}_{12}Mg$.

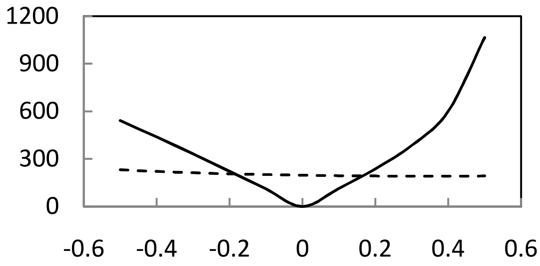

Fig. 5 The nucleus ${}^{28}_{14}Si$

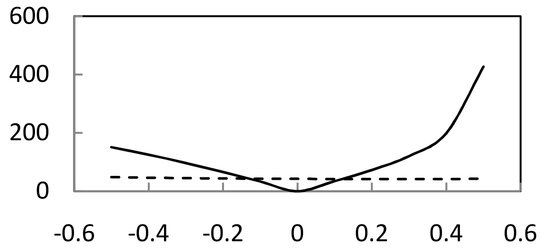

Fig. 6 The nucleus ${}^{32}_{16}S$

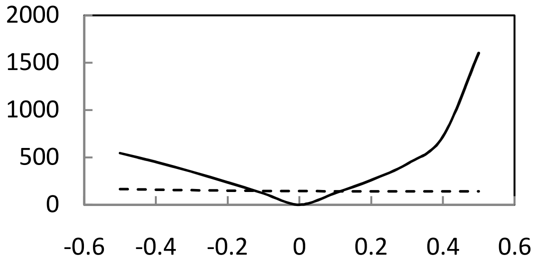

Fig. 7 The nucleus ${}^{36}_{18}Ar$

Figures-1, 2, 3, 4, 5, 6 and 7 Variation of the reciprocal cranking-model (solid) and rigid-body model (dashed) moments of inertia with respect to β for the nuclei ${}^{8}_{4}Be$, ${}^{12}_{6}C$, ${}^{20}_{10}Ne$, ${}^{24}_{12}Mg$,${}^{28}_{14}Si$, ${}^{32}_{16}S$ and ${}^{36}_{18}Ar$, respectively.

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