Code design of i-spotty-byte error correcting codes

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Abstract. Irregular-spotty-byte error control codes over the finite field $\mathbf{F}_q$ devised by the author [2] are matrix codes which generalizes the usual spotty-byte-codes [5]. Here a word is divided into irregular bytes of different lengths and distance between distinct words is measured in terms of newly defined i-spotty-byte metric function [2,3]. In this paper, we present the code construction methods of the i-spotty-byte error correcting codes in terms of their parity check matrix.

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1. Introduction

Spotty-byte error control codes devised by Suzuki et.al.[5,6,7] are matrix codes suitable for semi-conductor memories in which a word is divided into regular bytes of equal length "b". However, a more general and practical situation is when bytes are not regular i.e. when a word is divided into irregular bytes of different lengths. In a different setting, Feng et al [1] called such irregular bytes as *"blocks"* and studied error control codes endowed with the π-metric. In [2], the author introduced the notion of irregular-spotty-byte (or i-spotty-byte) error control codes generalizing the concept of both spotty-byte error control codes [5] and π-codes [1]. In [3], the author studied various weight enumerator polynomials of i-spotty-byte code viz. exact weight enumerator, complete weight enumerator, i-byte weight enumerator, i-spotty-byte weight enumerator and obtained the duality relations for them. In this paper, we present the code design methods of i-spotty-byte error correcting codes in terms of their parity check matrix.

We begin with the basic definitions and notations for i-spotty-byte error control codes [2].

2. Definitions and Notations

Let q be a prime or power of prime number. Let $\mathbf{F}_q$ be the finite field with q elements. A partition, P, of a positive integer N is defined as

$$P : N = n_1 + n_2 + \cdots + n_s, 1 \leq n_1 \leq n_2 \cdots \leq n_s \quad s \geq 1.$$

and is denoted as

$$P = [n_1][n_2] \cdots [n_s] = [m_1]^{l_1}[m_2]^{l_2} \cdots [m_r]^{l_r},$$

if

$$\begin{aligned}
&n_1 = n_2 = \cdots = n_{l_1} = m_1,\\
&n_{l_1+1} = n_{l_1+2} = \cdots = n_{l_1+l_2} = m_2,\\
&\vdots\\
&\vdots\\
&\vdots\\
&n_{l_1+l_2+\cdots+l_{r-1}+1} = n_{l_1+l_2+\cdots+l_{r-1}+2} = \cdots = n_{l_1+l_2+\cdots+l_r} = m_r.
\end{aligned}$$

Then we can write the field $\mathbf{F}_q^N$ as

$$\mathbf{F}_q^N = \mathbf{F}_q^{n_1} \oplus \mathbf{F}_q^{n_2} \oplus \cdots \oplus \mathbf{F}_q^{n_s}.$$

Each vector $v \in \mathbf{F}_q^N$ can be uniquely written as $v = (v_1, v_2, \cdots, v_s)$ where $v_i \in V_i = \mathbf{F}_q^{n_i}$ for all $1 \leq i \leq s$ and is called the i^{th} *irregular-byte* or simply i^{th} *i-byte of* v. We call the partition P as *primary partition* or *irregular-byte partition*. Further, let $1 \leq T \leq N$ be a positive integer and let $P' : T = [t_1][t_2] \cdots [t_s]$ be a partition of T where $1 \leq t_i \leq n_i$ for all $1 \leq i \leq s$. Then P' is called as *"secondary partition"* or *"error partition"*. Note that the secondary partition depends upon primary partition. The number N is called the *primary number* and T is called the *secondary number*.

Definition 2.1 [2]**.** Let N and T be the positive integers with $1 \leq T \leq N$. Let P and P' be the primary and secondary partitions corresponding to N

and T respectively given by

$$\begin{array}{rcl} P:N & = & [n_1][n_2]\cdots[n_s], \\ \text{and} & & \\ P':T & = & [t_1][t_2]\cdots[t_s], \end{array}$$

where $1 \leq t_i \leq n_i$ for all $1 \leq i \leq s$.

Let u be a vector in $\mathbf{F}_q^N = \oplus_{i=1}\mathbf{F}_q^{n_i}$ given by $u = (u_1, u_2, \cdots, u_s)$ where $u_i \in \mathbf{F}_q^{n_i}$ for all i is the i^{th} i-byte of u of size n_i. We define the *irregular-spotty weight* (or simply *i-spotty weight*) $w_\beta^{(P,P')}(u)$ of u corresponding to the primary-partition P and secondary-partition P' as

$$w_\beta^{(P,P')}(u) = \sum_{i=1}^{s}\left\lceil \frac{w_H(u_i)}{t_i}\right\rceil,$$

where $w_H(u_i)$ is the Hamming weight of the i^{th} i-byte u_i of size n_i and $\lceil x \rceil$ denotes the smallest integer greater than or equal to x.

Definition 2.2 [2]**.** The irregular-spotty distance (or simply *i-spotty distance*) between two vectors $u = (u_1, u_2, \cdots, u_s)$ and $v = (v_1, v_2, \cdots v_s)$ in $\mathbf{F}_q^N = \oplus_{i=1}^{s}\mathbf{F}_q^{n_i}$ is given by

$$\begin{array}{rcl} d_\beta^{(P,P')}(u,v) & = & w_\beta^{(P,P')}(u-v) \\ & = & \displaystyle\sum_{i=1}^{s}\left\lceil \frac{d_H(u_i,v_i)}{t_i}\right\rceil, \end{array}$$

where $d_H(u_i, v_i)$ is the Hamming distance between the i^{th} i-bytes u_i and v_i of u and v respectively. Then i-spotty distance is a metric function on $\mathbf{F}_q^N = \oplus_{i=1}^{s}\mathbf{F}_q^{n_i}$.

Note. We also call i-spotty weight and i-spotty distance as "t_i/n_i-weight" and "t_i/n_i-distance" respectively. Moreover, we simply denote the i-spotty weight $w_\beta^{(P,P')}$ and i-spotty distance $d_\beta^{(P,P')}$ by w_β and d_β respectively when the primary partition P and secondary partition P' are clear from the context.

Observations.

(i) Let t, s and b be positive integers with $1 \leq t \leq b$. Taking $N = bs, T = ts$, $n_i = b$ and $t_i = t$ for all i, then i-spotty distance (weight) reduces to the spotty-distance (weight) introduced by Suzuki et al. [5].

(ii) If $t_i = 1$ for all $1 \leq i \leq s$, then $w_\beta(x)$ where $x = (x_1, x_2, \cdots, x_s) \in \mathbf{F}_q^N = \oplus_{i=1}^s \mathbf{F}_q^{n_i}$ is expressed as

$$\begin{aligned} w_\beta(x) &= \sum_{i=1}^{s} \left\lceil \frac{w_H(x_i)}{1} \right\rceil \\ &= \sum_{i=1}^{s} w_H(x_i) \\ &= \text{Hamming weight of } x. \end{aligned}$$

(iii) If $t_i = n_i$ for all $1 \leq i \leq s$ i.e. when secondary partition P' is equal to the primary partition P, then $w_\beta(x)$ for $x = (x_1, \cdots, x_s) \in \mathbf{F}_q^N = \oplus_{i=1}^s \mathbf{F}_q^{n_i}$ is expressed as

$$w_\beta(x) = \sum_{i=1}^{s} \left\lceil \frac{w_H(x_i)}{n_i} \right\rceil.$$

Here

$$\left\lceil \frac{w_H(x_i)}{n_i} \right\rceil = \begin{cases} 0 & \text{if} \quad w_H(x_i) = 0, \\ 1 & \text{if} \quad w_H(x_i) \neq 0. \end{cases}$$

Thus

$$\begin{aligned} w_\beta(x) &= \# \{ i \mid i \ \leq i \leq s, x_i \neq 0 \} \\ &= \pi\text{-weight } of \ x \quad [1]. \end{aligned}$$

(iv) Let $x = (x_1, x_2, \cdots, x_s) \in \mathbf{F}_q^N = \oplus_{i=1}^s \mathbf{F}_q^{n_i}$. If $w_\beta(x) = \sum_{i=1}^{s} \left\lceil \frac{w_H(x_i)}{t_i} \right\rceil = \mu$ then we say that *i-spotty weight* or *i-spotty measure* of x is μ. Equivalently, we also say that t_i/n_i-measure of x is μ.

(v) Let $b_i = \left\lceil \frac{n_i}{t_i} \right\rceil$ for all $1 \leq i \leq s$. Then b_i is the maximum number of t_i/n_i-errors (or i-spotty errors) that can occur in the i^{th} i-byte of size n_i. Let $\hat{b} = \sum_{i=1}^{s} b_i$. Then $\hat{b}$ is the maximum number of t_i/n_i-errors (or i-spotty errors) that can occur in a word $x = (x_1, x_2, \cdots x_s) \in \oplus_{i=1}^{s} \mathbf{F}_q^{n_i} = \mathbf{F}_q^N$.

(vi) Let $\theta_Z(x)$ be the total number of (erroneous) i-bytes in a word $x \in \oplus_{i=1}^{s} \mathbf{F}_q^{n_i} = \mathbf{F}_q^N$ having Z number of t_i/n_i-errors where $Z = 0, 1, 2, \cdots, b$; $b = \max_{i=1}^{s}\{b_i\}$ and $b_i's$ are as given in (v).

Let

$$\sigma = \theta_1(x) + \theta_2(x) \cdots + \theta_b(x)$$
$$= \text{total number of erroneous i-bytes } in\ x.$$

Then the total number of i-bytes in the word x is expressed as

$$s = \sigma + \theta_0(x)$$
$$= \theta_0(x) + \theta_1(x) + \cdots + \theta_b(x).$$

Using these functions $\theta_Z{}'s$, the i-spotty weight (or i-spotty measure) of $x \in \oplus_{i=1}^{s} \mathbf{F}_q^{n_i} = \mathbf{F}_q^N$ is expressed as

$$w_\beta(x) = \theta_1(x) + 2\theta_2(x) + \cdots + b\theta_b(x),$$

where

$$b = \max_{i=1}^{s}\{b_i\} = \max_{i=1}^{s}\left\{\left\lceil \frac{n_i}{t_i} \right\rceil\right\}.$$

Definition 2.3 [2]. Let T and N be positive integers with $1 \leq T \leq N$. Let $V \subseteq \mathbf{F}_q^N = \oplus_{i=1}^{s} \mathbf{F}_q^{n_i}$ be an $\mathbf{F}_q$-subspace of $\mathbf{F}_q^N = \oplus_{i=1}^{s} \mathbf{F}_q^{n_i}$ equipped with the i-spotty metric d_β corresponding to the primary partition P of N and secondary partition P' of T. Then V is called an *irregular-spotty-byte* (or simply *i-spotty-byte*) error control code and is denoted by

$[N, k, d_\beta; P, P']$ where $P : N = [n_1][n_2]\cdots[n_s]$ is the irregular-byte partition, $P' : T = [t_1][t_2]\cdots[t_s]$, $1 \leq t_i \leq n_i$ is the error partition, $k = \dim_{\mathbf{F}_q} V$ and $d_\beta =$ minimum i-spotty distance of $V = \min\limits_{\substack{x,y\in V\\ x\neq y}} d_\beta(x, y)$.

3. Code design of i-spotty-byte error correcting codes

In this section, we first give the code construction method of i-spotty-byte codes correcting all i-spotty-byte errors of measure 1 and then generalize the method for the construction of codes correcting all i-spotty-byte errors of measure μ or less $(\mu \geq 1)$.

We begin with few definitions:

Definition 3.1 [5]**.** Given a monic primitive polynomial $g(x)$ of degree r over $\mathbf{F}_q$, the $r \times r$ companion matrix M corresponding to $g(x)$ is defined as follows:

$$g(x) = g_0 + g_1 x + g_2 x^2 + \cdots\cdots + g_{r-2}x^{r-2} + g_{r-1}x^{r-1} + x^r,$$

$$M = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 & -g_0 \\ 1 & 0 & \cdots & 0 & 0 & -g_1 \\ 0 & 1 & \cdots & 0 & 0 & -g_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & -g_{r-2} \\ 0 & 0 & \cdots & 0 & 1 & -g_{r-1} \end{pmatrix}_{r\times r}$$

Observations.

(i) Let α be a primitive element of $\mathbf{F}_q^r$ and a root of $g(x)$. Its companion matrix M has its columns $\begin{pmatrix} \vdots \\ \vdots \\ \alpha^i \\ \vdots \\ \vdots \end{pmatrix}$ for $i = 1$ to r where $\begin{pmatrix} \vdots \\ \vdots \\ \alpha^i \\ \vdots \\ \vdots \end{pmatrix}$ is the coefficient vector of $x^i (mod\ g(x))$.

The companion matrix of α^j is M^j and its column vectors are expressed as follows:

$$M^j = \begin{pmatrix} \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ \alpha^j & \alpha^{j+1} & \cdots & \alpha^{j+r-1} \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \end{pmatrix}_{r\times r}.$$

Let e be the exponent of $g(x)$, that is, $y = e$ is the least positive solution of $x^y \equiv (mod\ g(x))$. The companion matrix M has the following properties [5]:

(a) M is non singular.

(b) $M^0 = M^e = I_r$.

(c) $M^i = M^j$ if and only if $i \equiv j(mod\ e)$.

Definition 3.2. Let $1 \leq n_1 \leq n_2 \cdots \leq n_s$ and $1 \leq t_1 \leq t_2 \cdots t_2$ be positive integer with $1 \leq t_i \leq n_i$ for all $1 \leq i \leq s,\ s \geq 1$. Let l and r be positive integers such that

$$l \geq 2(\max_{i=1}^{s}\{t_i\}) \text{ and } \quad r \geq \max_{i=i}^{s}\{t_i\}.$$

Further, for $i =$ to s, let

(i) $H_i' = [h_{i,1}', h_{i,2}', \cdots, h_{i,n_i}'],\ h_{i,k}' \in \mathbf{F}_q{}^l$ for all $1 \leq k \leq n_i$, be $l \times n_i$ matrices over $\mathbf{F}_q$ satisfying the following two properties:

  (a) Every set of $2t_i$ (or fewer) columns of H_i' are linearly independent over $\mathbf{F}_q$; and

  (b) Every set of (t_i+t_j) (or fewer) columns with t_i (or fewer) columns taken from H_i' and t_j (or fewer) columns taken from H_j' $(i \neq j)$ are linearly independent over $\mathbf{F}_q$.

(ii) $H_i'' = [h_{i,1}'', h_{i,2}'', \cdots, h_{i,n_i}'']$, $h_{i,k}'' \in \mathbf{F}_q{}^r$ for all $1 \leq k \leq n_i$, be $r \times n_i$ matrices over $\mathbf{F}_q$ such that every set of t_i (or fewer) columns of H_i'' are linearly independent over $\mathbf{F}_q$.

Theorem 3.3. *Using the notations as given in Definition 3.2, let M be an $r \times r$ companion matrix over $\mathbf{F}_q$. Let $m = q^r - 1$. The null space of $H = [H_1, H_2, \cdots, H_s]$ where each H_i $(1 \le i \le s)$ is a $(l + r) \times mn_i$ submatrix given by*

$$H_i = \begin{pmatrix} H_i' & H_i' & \cdots & H_i' \\ M^0 H_i'' & M^1 H_i'' & \cdots & M^{(m-1)} H_i'' \end{pmatrix}_{(l+r)\times mn_i}.$$

is a single t_i/n_i-error correcting code ($S_{t_i/n_i}EC$) with check bit length $R = l + r$ and code length $N = mn = (q^r - 1)n$ where $n = n_1 + n_2 + \cdots n_s$. The parameters of the resulting i-spotty-byte code will be

$$[(q^r - 1)n, (q^r - 1)n - (l + r), 3\ ; P, P'],$$

where

$$\begin{aligned} P : N = (q^r - 1)n &= [n_1]^m [n_2]^m \cdots [n_s]^m \\ &= [n_1]^{(q^{r-1})} [n_2]^{(q^{r-1})} \cdots [n_s]^{(q^{r-1})}, \end{aligned}$$

and

$$\begin{aligned} P' : T = (q^r - 1)t &= [t_1]^m [t_2]^m \cdots [t_s]^m \\ &= [t_1]^{(q^{r-1})} [t_2]^{(q^{r-1})} \cdots [t_s]^{(q^{r-1})}, \end{aligned}$$

and

$$t = t_1 + t_2 + \cdots + t_s.$$

Proof. For each $i = 1$ to s, let

$$\begin{aligned} E_{t_i/n_i} &= \{e = (e_1^0, e_1^1, \cdots, e_1^{m-1}, \cdots\cdots, e_s^0, e_s^1, \cdots, e_s^{m-1} \mid e_p^u \in \mathbf{F}_q^{n_p} \\ &\quad \text{for all } 0 \le u \le m-1, 1 \le p \le s \text{ and } 1 \le w_H(e_p^u) \le t_i \\ &\quad \text{for } p = i \text{ and for exactly one value of } u \text{ and } w_H(e_p^u) = 0 \\ &\quad \text{otherwise}\} \\ &= \text{set of all single } t_i/n_i - \text{errors occuring in the } i^{th} \text{ i-byte.} \end{aligned}$$

Let $E = \cup_{1=1}^{s} E_{t_i/n_i}$=collection of all single t_i/n_i-errors.

Given $H = [H_1, H_2, \cdots, H_s]$ where each $H_i (1 \leq i \leq s)$ contains m i-bytes each of size n_i. We call H_i as the i^{th} *sector* of H of size mn_i. The j^{th} i-byte $(0 \leq j \leq m-1)$ in the i^{th} sector H_i is given by

$$\begin{pmatrix} H'_i \\ M^j H''_i \end{pmatrix}.$$

To prove the theorem, it suffices to show that

(i) $eH^T \neq 0$ for all $e \in E$, and

(ii) $eH^T \neq e'H^T$ for all $e, e' \in E, e \neq e'$.

Proof of (i). Let $e \in E$. Then $e \in E_{t_i/n_i}$ for some i. This means that e is of the form

$$e = (0, \cdots, 0, e_i^j, 0, \cdots 0),$$

where $e_i^j \in \mathbf{F}_q^{n_i}, 0 \leq j \leq m-1$ and $1 \leq w_H(e_i^j) \leq t_i$.

Let if possible $eH^T = 0$. Then we have,

$$\begin{aligned} e_i^j \begin{pmatrix} H'_i \\ M^j H''_i \end{pmatrix}^T &= 0. \\ \Rightarrow e_i^j {H'_i}^T &= 0. \end{aligned}$$

The above equation gives $e_i^j = (0, 0, \cdots, 0)$ as every set of $2t_i$ (or fewer) columns of H'_i are linearly independent over $\mathbf{F}_q$. A contradiction, Hence $eH^T \neq 0$ for all $e \in E$.

Proof of (ii). Let $e, e' \in E$ with $e \neq e'$. Then $e \in E_{t_i/n_i}$ and $e' \in E_{t_k/n_k}$ for some i and k. Let

$$e = (0, \cdots, 0, e_i^j, 0, \cdots 0),$$

where $e_i^j \in \mathbf{F}_q^{n_i}, 0 \leq j \leq m-1$ and $1 \leq w_H(e_i^j) \leq t_i$, and

$$e' = (0, \cdots, 0, f_k^p, 0 \cdots 0)$$

where $f_k^p \in \mathbf{F}_q^{n_k}, 0 \leq p \leq m-1$ and $1 \leq w_H(f_k^p) \leq t_k$.

Let if possible $eH^T = e'H^T$. There are two cases to consider depending on i and k:

Case 1. When $i = k$.

In this case e and e' are of the form

$$e = (0, \cdots, 0, e_i^j, 0, \cdots 0),$$

and

$$e' = (0, \cdots, 0, f_i^p, 0 \cdots 0),$$

where $e_i^j, f_i^p \in \mathbf{F}_q^{n_i}, 0 \leq j, p \leq m-1$ and $1 \leq w_H(e_i^j), w_H(f_i^p) \leq t_i$.

In this case, there are two subcases to consider:

Subcase 1. When $j = p$.

In this subcase, e and e' are of the form

$$e = (0, \cdots, 0, e_i^j, 0, \cdots 0),$$

and

$$e' = (0, \cdots, 0, f_i^j, 0 \cdots 0).$$

Also, $eH^T = e'H^T$ gives

$$e_i^j \begin{pmatrix} H_i' \\ M^j H_i'' \end{pmatrix}^T = f_i^j \begin{pmatrix} H_i' \\ M^j H_i'' \end{pmatrix}^T.$$

which implies

$$(e_i^j - f_i^j)H_i'^T = 0.$$

The above equation gives $(e_i^j - f_i^j) = 0$ because every set of $2t_i$ (or fewer) columns of H_i' are linearly independently over $\mathbf{F}_q$ and $1 \leq w_H(e_i^j), w_H(f_i^j) \leq t_i$. Thus, we have $e_i^j = f_i^j$ which means $e = e'$. A contradiction.

Subcase 2. When $j \neq p$.

Then $eH^T = e'H^T$ gives

$$e_i^j \begin{pmatrix} H_i' \\ M^j H_i'' \end{pmatrix}^T = f_i^p \begin{pmatrix} H_i' \\ M^p H_i'' \end{pmatrix}^T$$

$$\Rightarrow (e_i^j - f_i^p)H_i'^T = 0$$

$$\Rightarrow (e_i^j - f_i^p) = 0,$$

as every set of $2t_i$ or fewer columns of H'_i are linearly independent over $\mathbf{F}_q$. This gives $e_i^j = f_i^p$ and hence $e = e'$. A contradiction.

Case 2. When $i \neq k$.

In this case, again we have two subcases to consider:

Subcase 1. When $j = p$.

In this subcase $eH^T = e'H^T$ gives

$$e_i^j \begin{pmatrix} H'_i \\ M^j H''_i \end{pmatrix}^T = f_k^j \begin{pmatrix} H'_k \\ M^j H''_k \end{pmatrix}^T$$
$$\Rightarrow e_i^j H_i'^T - f_k^j H_k'^T = 0.$$

The above equation gives $e_i^j = (0, \cdots 0)_{1 \times n_i}$ and $f_k^j = (0, \cdots, 0)_{1 \times n_k}$ as by assumption every set of $(t_i + t_k)$ (or fewer) columns with t_i (or fewer) columns taken from H'_i and t_k (or fewer) columns taken from H'_k are linearly independent over $\mathbf{F}_q$. Thus we have

$$e = e' = (0, \cdots, 0). \text{ A contradiction.}$$

Subcase 2. When $j \neq p$.

In this subcase again $eH^T = e'H^T$ gives

$$e_i^j \begin{pmatrix} H'_i \\ M^j H''_i \end{pmatrix}^T = f_k^p \begin{pmatrix} H'_k \\ M^p H''_k \end{pmatrix}^T.$$

This implies that

$$e_i^j H_i'^T - f_k^p H_k'^T = 0,$$

where

$$1 \leq w_H(e_i^j) \leq t_i,$$
$$1 \leq w_H(f_k^p)] \leq t_k,$$

which again gives

$$e_i^j = (0, \cdots, 0)_{1 \times n_i} \quad \text{and } f_k^p = (0, \cdots, 0)_{1 \times n_k}$$

by the same argument as given in Subcase 1. A contradiction again.

Combining the two cases, we get

$$eH^T \neq e'H^T \text{for all } e, e' \in E, e \neq e'.$$

Hence the theorem. □

Remark 3.4. We may also construct the shortened version of the i-spotty code constructed in Theorem 3.3 by taking $P : N' = [n_1]^{m_1}[n_2]^{m_2}\cdots[n_s]^{m_s}$ and $P' : T' = [t_1]^{m_1}[t_2]^{m_2}\cdots[t_s]^{m_s}$ where $m_i \leq m = q^r - 1$ for all $1 \leq i \leq s$ and keeping only the first m_i i-bytes in the i^{th} sector H_i of the parity check matrix H. For example, if $m_1 = 1, m_2 = 2, \cdots, m_s = s$ where $s \leq q^r - 1$, then we can take the parity check matrix of the single i-spotty-byte error correcting code as

$$H = \begin{pmatrix} H_1' & \vdots & H_2' & H_2' & \vdots & \cdots & \vdots & H_s' & \cdots & H_s' \\ M^0H_1' & \vdots & M^0H_2'' & M'H_2'' & \vdots & \cdots & \vdots & M^0H_s'' & \cdots & M^{s-1}H_s'' \end{pmatrix}$$

We can generalize the result of Theorem 3.3 for the design of i-spotty-byte code correcting all t_i/n_i-errors of measure μ or less ($\mu \geq 1$). For this, we begin with the following definitions:

Definition 3.5. Let $\mu, 1 \leq n_1 \leq n_2 \leq \cdots \leq n_s$ and $1 \leq t_1 \leq t_2 \leq \cdots \leq t_s$ be positive integers with $1 \leq t_i \leq n_i$ for all $1 \leq i \leq s$. Let l and r be the positive integers such that

$$l \geq \max_{i=1}^{s}\{2\mu t_i\} \quad \text{and} \quad r \geq \max_{i=1}^{s}\{\mu t_i\}.$$

Further, for $i = 1$ to s, let

(i) $H_i' = [h_{i,1}', h_{i,2}' \cdots h_{i,n_i}'], h_{i,k}' \in \mathbf{F}_q^l$ for all $1 \leq k \leq n_i$, be $l \times n_i$ matrices over $\mathbf{F}_q$ satisfying the following two properties:

(a) Every set of $2\mu t_i$ (or fewer) columns of H_i' are linearly independent over $\mathbf{F}_q$.

(b) If $j_1, j_2, \cdots, j_s$ are nonnegative integers such that $0 \leq j_i \leq n_i$ for all $i = 1$ to s satisfying

$$\left\lceil \frac{j_1}{t_1} \right\rceil + \left\lceil \frac{j_2}{t_2} \right\rceil + \cdots\cdots + \left\lceil \frac{j_s}{t_s} \right\rceil \leq 2\mu,$$

then every set of $(j_1 + j_2 + \cdots + j_s)$ (or fewer) columns with j_i columns taken from $H'_i(i = 1 \text{ to } s)$ are linearly independent over $\mathbf{F}_q$. Here the symbol $\lceil x \rceil$ denotes the smallest integer greater than or equal to x.

(ii) $H''_i = [h''_{i,1}, h''_{i,2} \cdots h''_{i,n_i}], h''_{i,j} \in \mathbf{F}_q^r$ for all $1 \leq j \leq n_i$, be $r \times n_i$ matrices over $\mathbf{F}_q$ such that every set of μt_i (or fewer) columns of H''_i are linearly independent over $\mathbf{F}_q$.

Theorem 3.6. *Using the notations as given in Definitions 3.5, let M be an $r \times r$ companion matrix over $\mathbf{F}_q$. Let $m = q^r - 1$. The null space of $H = [H_1, H_2, \cdots, H_s]$, where each $H_i(1 \leq i \leq s)$ is a $(l + (2\mu - 1)r) \times mn_i$ submatrix given by*

$$H_i = \begin{pmatrix} H'_i & H'_i & H'_i & \cdots & H'_i \\ M^0 H''_i & M^1 H''_i & M^2 H''_i & \vdots & M^{(m-1)} H''_i \\ M^0 H''_i & M^2 H''_i & M^4 H''_i & \vdots & M^{2(m-1)} H''_i \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ M^0 H''_i & M^{(2\mu-1)} H''_i & M^{2(2\mu-1)} H''_i & \cdots & M^{(2\mu-1)(m-1)} H''_i \end{pmatrix}_{(l+(2\mu-1)r)\times mn_i}$$

is an i-spotty-byte error control code V correcting all t_i/n_i-errors of measure μ (or less) and having check but length $R = l + (2\mu - 1)r$ and code length $N = mn = (q^r - 1)n$ where $n = n_1 + n_2 + \cdots n_s$. The parameters of the resulting code will be

$$[mn, mn - (l + (2\mu - 1)r), (2\mu + 1); P, P'],$$

where $P : N = mn = [n_1]^m [n_2]^m \cdots [n_s]^m$ and *$P' : T = mt = [t_1]^m [t_2]^m \cdots \cdots [t_s]^m, t = t_1 + t_2 + \cdots + t_s$.*

Proof. It suffices to prove that the code V which is the null space of H detects all i-spotty-byte errors of measure 2μ or less meaning thereby that the minimum i-spotty distance of the code is atleast $2\mu + 1$.

Let $e \in \mathbf{F}_q^N = \mathbf{F}_q^{m(n_1+\cdots+n_s)}$ with $w_\beta(e) \leq 2\mu$.

Then e is of the form

$$e = (e_1 \cdots e_s) = (e_1^0, e_1^1, \cdots, e_1^{m-1}, \cdots, e_s^0, e_s^1, \cdots e_s^{m-1}),$$

where e_j is the j^{th} sector of e and $e_p^u \in \mathbf{F}_q^{n_p}$ for all $0 \leq u \leq m-1, 1 \leq p \leq s$, and

$$\sum_{p=1}^{s}\sum_{u=0}^{m-1}\left\lceil \frac{w_H(e_p^u)}{t_p} \right\rceil \leq 2\mu.$$

We claim that $eH^T \neq 0$.

Let σ be the total number of erroneous sectors in e. There are two cases to consider:

Case 1. When $\sigma = 1$.

Let j^{th} sector in e is in error having erroneous i-bytes say $e_j^{u_1}, e_j^{u_2}, \cdots, e_j^{u_{j^*}}$ with

$$\sum_{k=1}^{u_{j^*}}\left\lceil \frac{w_H(e_j^{u_k})}{t_j} \right\rceil \leq 2\mu.$$

Then the Hamming weight of the j^{th} sector $e_j = (e_j^0, e_j^1, \cdots e_j^{m-1})$ in e is les than or equal to $2\mu t_j$. Since H_j' is an $l \times n_j$ q-ary matrix whose every set of $2\mu t_j$ (or fewer) columns are linearly independent over $\mathbf{F}_q$. Therefore, we must have $eH^T \neq 0$.

Case 2. When $\sigma \geq 2$.

Let if possible $eH^T = 0$. Let us assume that $e_j, e_k, \cdots, e_y$ be the erroneous sectors in e such that $e_j^{u_1}, e_j^{u_2}, \cdots, e_j^{u_{j^*}}$ be the erroneous i-bytes in e_j; $e_k^{v_1}, e_k^{v_2}, \cdots, e_k^{v_{k^*}}$ be the erroneous i-bytes in e_k; $\cdots\cdots e_y^{\theta_1}, e_y^{\theta_2}, \cdots, e_y^{\theta_{y^*}}$ be the erroneous i-bytes in e_y; where

$$\sum_{\pi=j,k,\cdots,y}\ \sum_{\lambda=u_1\cdots u_{j^*},v_1\cdots v_{k^*}\cdots\theta_1,\cdots,\theta_{y^*}}\left\lceil \frac{w_H(e_\pi^\lambda}{t_\pi}) \right\rceil \leq 2\mu,$$

and

$$0 \leq u_1, u_2, \cdots, u_{j^*}, v_1, \cdots, v_{k^*}, \cdots, \theta_1, \cdots, \theta_{y^*} \leq m-1.$$

Then $eH^T = 0$ gives the following relation:

$$\begin{aligned} & e_j^{u_1}\left[H_j'^T \quad (M^{u_1}H_j'')^T \quad (M^{2u_1}H_j'')^T \cdots (M^{(2\mu-1)u_1}H_j'')^T\right] \\ & +e_j^{u_2}\left[H_j'^T \quad (M^{u_2}H_j'')^T \quad (M^{2u_2}H_j'')^T \cdots (M^{(2\mu-1)u_2}H_j'')^T\right] \\ & +\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \end{aligned}$$

$$
\begin{aligned}
&+e_j^{u_{j*}} \left[H_j'^T \; (M_{j*}^u H_j'')^T \quad (M^{(2\mu-1)u_1} H_j'')^T \cdots (M^{(2\mu-1)u_j} H_j'')^T\right] \\
&+e_k^{v_1}\left[H_k'^T \quad (M^{v_1} H_k'')^T \quad (M^{2v_1} H_k'')^T \cdots (M^{(2\mu-1)v_1} H_k'')^T\right] \\
&+\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&+e_k^{v_{k*}}\left[H_k'^T \quad (M^{v_{k*}} H_j'')^T \quad (M^{(2v_{k*}} H_k'')^T \cdots (M^{(2\mu-1)v_{k*}} H_j'')^T\right] \quad (1) \\
&+\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&+e_y^{\theta_1}\left[H_y'^T \quad (M^{\theta_1} H_y'')^T \quad (M^{2\theta_1} H_y'')^T \cdots (M^{(2\mu-1)\theta_1} H_y'')^T\right] \\
&+\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&+e_y^{\theta_{y*}}\left[H_y'^T \quad (M^{\theta_{y*}} H_y'')^T \quad (M^{(2\theta_{y*}} H_y'')^T \cdots (M^{(2\mu-1)\theta_{y*}} H_y'')^T\right] \\
= \;& [O_l \quad O_r \quad O_r \cdots\cdots O_r],
\end{aligned}
$$

where O_l and O_r are the $1 \times l$ and $1 \times r$ null matrices over $\mathbf{F}_q$ respectively.

The relation

$$\left(\sum_{\rho=u_1}^{u_{j*}} e_j^\rho\right) H_j'^T + \left(\sum_{w=v_1}^{v_k^*} e_k^w\right) H_k'^T + \cdots + \left(\sum_{f=\theta_1}^{\theta_{y*}} e_y^f\right) H_y'^T = O_l$$

leads to

$$\sum_{\rho=u_1}^{u_{j*}} e_j^\rho = O_{n_j}, \quad \sum_{w=v_1}^{v_{k*}} e_k^w = O_{n_k}, \cdots, \cdots \sum_{f=\theta_1}^{\theta_{y*}} e_y^f = O_{n_y},$$

because of property (i) (b) of Matrix H_i' given in Definition 3.5.

Multiplying the equation $\sum_{\rho=u_1}^{u_{j*}} e_j^\rho = O_{n_j}$ by $(H_j'')^T$, $\sum_{w=v_1}^{v_{k*}} e_k^w = O_{n_k}$ by $(H_k'')^T, \cdots$ $\cdots \sum_{f=\theta_1}^{\theta_{y*}} e_y^f = O_{n_y}$ by $(H_y'')^T$ from right gives

$$
\begin{aligned}
\left(\sum_{\rho=u_1}^{u_{j*}} e_j^\rho\right) H_j''^T &= O_r, \\
\left(\sum_{w=v_1}^{v_{k*}} e_k^w\right) H_k''^T &= O_r, \\
&\vdots
\end{aligned}
$$

$$\vdots$$
$$\vdots$$
$$\left(\sum_{f=\theta_1}^{\theta_{y^*}} e_y^f\right) H_y''^T = O_r.$$

The following equation from (1) is obtained:

$$\begin{aligned}
&\Big[(e_j^{u_1} H_j''^T)(M^{u_1})^T \cdots\cdots (e_j^{u_1} H_j''^T)(M^{(2\mu-1)u_1})^T\Big] \\
&+\Big[(e_j^{u_2} H_j''^T)(M^{u_2})^T \cdots\cdots (e_j^{u_2} H_j''^T)(M^{(2\mu-1)u_2})^T\Big] \\
&+\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&+\Big[(e_j^{u_{j^*}} H_j''^T)(M^{u_{j^*}})^T \cdots\cdots (e_j^{u_{j^*}} H_j''^T)(M^{(2\mu-1)u_{j^*}})^T\Big] \\
&+\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&+\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&+\Big[(e_y^{\theta_1} H_y''^T)(M^{\theta_1})^T \cdots\cdots (e_y^{\theta_1} H_y''^T(M^{(2\mu-1)\theta_1})^T\Big] \\
&+\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&+\Big[(e_y^{\theta_{y^*}} H_y''^T)(M^{\theta_{y^*}})^T \cdots\cdots (e_y^{\theta_{y^*}} H_y''^T)(M^{(2\mu-1)\theta_{y^*}})^T\Big] \\
&= [O_r \quad O_r \quad \cdots O_r].
\end{aligned} \tag{2}$$

Let $e_j^{u_1} H_j''^T, e_j^{u_2} H_j''^T \cdots e_j^{u_{j^*}} H_j''^T$ be denoted by $r_{u_1}, r_{u_2} \cdots r_{u_{j^*}}$ resp; $e_k^{v_1} H_k''^T$, $e_k^{v_2} H_k''^T \cdots e_k^{v_{k^*}} H_k''^T$ be denoted by $r_{v_1}, r_{v_2}, \cdots, r_{v_{k^*}}$ resp; $\cdots\cdots e_y^{\theta_1} H_y''^T, e_y^{\theta_2} H_y''^T$, $\cdots, e_y^{\theta_{y^*}} H_y''^T$ be denoted by $r_{\theta_1}, r_{\theta_2}, \cdots, r_{\theta_{y^*}}$ resp. Then (2) can be rewritten as

$$\begin{aligned}
&r_{u_1} + \cdots + r_{u_{j^*}} + r_{v_1} + \cdots + r_{v_{k^*}} + \cdots\cdots + r_{\theta_1} + \cdots + r_{\theta_{y^*}} = O_r \\
&r_{u_1}(M^{u_1})^T + \cdots + r_{u_{j^*}}(M^{u_{j^*}})^T + \cdots\cdots + r_{\theta_1}(M^{\theta_1})^T + \cdots\cdots \\
&+r_{\theta_{y^*}}(M^{\theta_{y^*}})^T = O_r \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&r_{u_1}(M^{(2\mu-1)u_1})^T + \cdots + r_{u_{j^*}}(M^{(2\mu-1)u_{j^*}})^T + \cdots\cdots + r_{\theta_1}(M^{(2\mu-1)\theta_1})^T \\
&+ \cdots + r_{\theta_{y^*}}(M^{(2\mu-1)\theta_{y^*}})^T = O_r.
\end{aligned} \tag{3}$$

Writing the above equation in the matrix form gives

$$(r_{u_1},\cdots,r_{u_{j^*}},\cdots,r_{\theta_1},\cdots,r_{\theta_{y^*}})\times$$

$$\times\begin{pmatrix} 1 & (M^{u_1})^T & \cdots & (M^{(2\mu-1)u_1})^T \\ \vdots & \vdots & \vdots & \vdots \\ 1 & (M^{u_{j^*}})^T & \cdots & (M^{(2\mu-1)(u_{j^*}})^T \\ \vdots & \vdots & \vdots & \vdots \\ 1 & (M^{\theta_1})^T & \cdots & (M^{(2\mu-1)\theta_1})^T \\ \vdots & \vdots & \vdots & \vdots \\ 1 & (M^{\theta_{y^*}})^T & \cdots & (M^{(2\mu-1)\theta_{y^*}})^T \end{pmatrix}$$

$$=(O_r \quad O_r \quad \cdots O_r),$$

or

$$(r_{u_1},\cdots,r_{u_{j^*}},\cdots,r_{\theta_1},\cdots,r_{\theta_{y^*}})\times$$

$$\times\begin{pmatrix} 1 & \cdots & 1 & \cdots & 1 & \cdots & 1 \\ M^{u_1} & \cdots & M^{u_{j^*}} & \cdots & M^{\theta_1} & \cdots & M_{\theta_{y^*}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ M^{(2\mu-1)u_1} & \cdots & M^{(2\mu-1)u_{j^*}} & \cdots & M^{(2\mu-1)\theta_1} & \cdots & M^{(2\mu-1)\theta_{y^*}} \end{pmatrix}^T$$

$$=(O_r \quad O_r \quad \cdots O_r).$$

Since the total numbers of erroneous i-bytes in all the erroneous sectors is $j^*+k^*+\cdots y^*=p+1$ (say) which is less than or equal to 2μ. therfore, writing the above matrix equation for the top $p+1(\leq 2\mu)$ relations, we get

$$(r_{u_1},\cdots,r_{u_{j^*}},\cdots,r_{\theta_1},\cdots,r_{\theta_{y^*}})\times$$

$$\times\begin{pmatrix} 1 & \cdots & 1 & \cdots & 1 & \cdots & 1 \\ M^{u_1} & \cdots & M^{u_{j^*}} & \cdots & M^{\theta_1} & \cdots & M_{\theta_{y^*}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ M^{pu_1} & \cdots & M^{pu_{j^*}} & \cdots & M^{p\theta_1} & \cdots & M^{p\theta_{y^*}} \end{pmatrix}^T$$

$$=(O_r \quad O_r \quad \cdots O_r).$$

The coefficient matrix in the above equation being Vandermonde's matrix is nonsingular. Therefore, relations (3) have a solution given by $r_{u_1} = \cdots = r_{u_{j^*}} = \cdots\cdots = r_{\theta_1} = \cdots = r_{\theta_{y^*}} = O_r$.

This implies that

$$e_j^{u_1} H_j''^T = e_j^{u_2} H_j''^T = e_j^{u_{j^*}} H_j''^T = \cdots e_y^{\theta_1} H_y''^T = \cdots = e_y^{\theta_{y^*}} H_y''^T = O_r$$

which further gives

$$e_j^{u_1} = e_j^{u_{j^*}} = O_{n_j}; \cdots; e_y^{\theta_1} = \cdots = e_y^{\theta_{y^*}} = O_{n_y},$$

as every μt_i columns of H_i'' are linearly independent over $\mathbf{F}_q$ for all $1 \le i \le s$. A contradiction. Hence $eH^T \neq 0$. □

Note. It is to be noted that there can exist atmost one erroneous i-byte in e having more than μ i-spotty errors. The fact is justified because if there are two or more i-bytes with more than μ i-spotty errors, then the total number of i-spotty errors in the error vector e will exceed 2μ which is a contradiction. In fact, if the i-spotty weight of an erroneous i-byte in e is more than μ, then any other erroneous i-byte will have i-spotty weight less than μ. That is why we need the condition that every set of μt_i(or fewer) columns of H_i'' are linearly independent over $\mathbf{F}_q$ in contrast to the condition of linear independence of $2\mu t_i$ columns as required in the case of matrices $H_i'(1 \le i \le s)$.

In the following example, we construct a single t_i/n_i-error correcting code.

Example 3.7. Let $q = 2, s = 3$ and $n_1 = n_2 = n_3 = t_1 = t_2 = t_3 = 2$.

Let $l = r = 4$. Then l and r satisfy

$$l \ge \max_{i=1}^{3}\{2t_i\} \text{ and } r \ge \max_{i=1}^{3}\{t_i\}.$$

Here $m = q^r - 1 = 2^4 - 1 = 15$. The code to be constructed as described in Theorem 3.3 will be a $[15 \times 6, (15 \times 6) - 8, 3; P, P'] = [90, 82, 3; P, P']$ i-spotty-byte code correcting all single t_i/n_i-errors where

$$P = P' : 90 = [2]^{15}[2]^{15}[2]^{15}.$$

However, we construct a shortend code of length 24 as discussed in Remark 3.4 by taking $m_1 = m_2 = m_3 = 4$ and

$$P = P' : 24 = [2]^4[2]^4[2]^4.$$

For this, let α be a root of $x^4 + x + 1 \in \mathbf{F}_2[x]$. Let M be the companion matrix of order 4×4 over $\mathbf{F}_2$ corresponding to α. Then

$$M^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}_{4\times 4}, \quad M^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}_{4\times 4},$$

$$M^2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}_{4\times 4}, \quad M^3 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}_{4\times 4}.$$

Let

$$H_1' = \begin{pmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}_{4\times 2} \quad H_2' = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}_{4\times 2} \quad H_3' = \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}_{4\times 2},$$

and

$$H_1'' = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{pmatrix}_{4\times 2} \quad H_2'' = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}_{4\times 2} \quad H_3'' = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}_{4\times 2}.$$

Then

$$H = \begin{pmatrix} H_1' & H_1' & H_1' & H_1' & \vdots & H_2' & H_2' & H_2' & H_2' & \vdots & H_3' & H_3' & H_3' & H_3' \\ M^0H_1'' & M^1H_1'' & M^2H_1'' & M^3H_1'' & \vdots & M^0H_2'' & M^1H_2'' & M^2H_2'' & M^3H_2'' & \vdots & M^0H_3'' & M^1H_3'' & M^2H_3'' & M^3H_3'' \end{pmatrix}$$

$$= \begin{pmatrix} 01 & 01 & 01 & 01 & \vdots & 10 & 10 & 10 & 10 & \vdots & 01 & 01 & 01 & 01 \\ 00 & 00 & 00 & 00 & \vdots & 01 & 01 & 01 & 01 & \vdots & 01 & 01 & 01 & 01 \\ 10 & 10 & 10 & 10 & \vdots & 00 & 00 & 00 & 00 & \vdots & 01 & 01 & 01 & 01 \\ 01 & 01 & 01 & 01 & \vdots & 00 & 00 & 00 & 00 & \vdots & 11 & 11 & 11 & 11 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 11 & 00 & 10 & 10 & \vdots & 01 & 00 & 01 & 01 & \vdots & 00 & 01 & 00 & 10 \\ 10 & 01 & 00 & 00 & \vdots & 01 & 11 & 01 & 00 & \vdots & 10 & 01 & 01 & 10 \\ 10 & 00 & 01 & 00 & \vdots & 01 & 01 & 11 & 01 & \vdots & 00 & 10 & 01 & 01 \\ 10 & 10 & 10 & 01 & \vdots & 00 & 01 & 01 & 11 & \vdots & 01 & 00 & 10 & 01 \end{pmatrix}_{8\times 24}$$

is the parity check matrix of an $[N, N-R; P, P']$ single t_i/n_i-error correcting code where $N = 24$ and $R = 8$. The fact that the code which is the null space of H is a single t_i/n_i-error correcting code is justified by Table 3.1 which shows that syndrome of all single t_i/n_i-errors are all distinct.

Table 3.1

Error Patterns of i-spotty-byte measure 1	Syndromes
(10⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00)	(0010⋮1111)
(01⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00)	(1001⋮1000)
(11⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00)	(1011⋮0111)
(00⋮10⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00)	(0010⋮0001)
(00⋮01⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00)	(1001⋮0100)
(00⋮11⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00)	(1011⋮0101)
(00⋮00⋮10⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00)	(0010⋮1001)
(00⋮00⋮01⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00)	(1001⋮0010)
(00⋮00⋮11⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00)	(1011⋮1011)
(00⋮00⋮00⋮10⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00)	(0010⋮1000)
(00⋮00⋮00⋮01⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00)	(1001⋮0001)
(00⋮00⋮00⋮11⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00)	(1011⋮1001)
(00⋮00⋮00⋮00⋮10⋮00⋮00⋮00⋮00⋮00⋮00⋮00)	(1000⋮1000)
(00⋮00⋮00⋮00⋮01⋮00⋮00⋮00⋮00⋮00⋮00⋮00)	(0100⋮1110)
(00⋮00⋮00⋮00⋮11⋮00⋮00⋮00⋮00⋮00⋮00⋮00)	(1100⋮0110)
(00⋮00⋮00⋮00⋮00⋮10⋮00⋮00⋮00⋮00⋮00⋮00)	(1000⋮0100)
(00⋮00⋮00⋮00⋮00⋮01⋮00⋮00⋮00⋮00⋮00⋮00)	(0100⋮0111)
(00⋮00⋮00⋮00⋮00⋮11⋮00⋮00⋮00⋮00⋮00⋮00)	(1100⋮0011)
(00⋮00⋮00⋮00⋮00⋮00⋮10⋮00⋮00⋮00⋮00⋮00)	(1000⋮0010)
(00⋮00⋮00⋮00⋮00⋮00⋮01⋮00⋮00⋮00⋮00⋮00)	(0100⋮1111)
(00⋮00⋮00⋮00⋮00⋮00⋮11⋮00⋮00⋮00⋮00⋮00)	(1100⋮1101)
(00⋮00⋮00⋮00⋮00⋮00⋮00⋮10⋮00⋮00⋮00⋮00)	(1000⋮0001)

Table contd.

Error Patterns of i-spotty-byte measure 1	Syndromes
(00⋮00⋮00⋮00⋮00⋮00⋮00⋮01⋮00⋮00⋮00⋮00)	(0100⋮1011)
(00⋮00⋮00⋮00⋮00⋮00⋮00⋮11⋮00⋮00⋮00⋮00)	(1100⋮1010)
(00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮10⋮00⋮00⋮00)	(1000⋮0100)
(00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮01⋮00⋮00⋮00)	(0100⋮0001)
(00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮11⋮00⋮00⋮00)	(1100⋮0101)
(00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮10⋮00⋮00)	(0001⋮0010)
(00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮01⋮00⋮00)	(1111⋮1100)
(00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮11⋮00⋮00)	(1110⋮1110)
(00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮10⋮00)	(0001⋮0001)
(00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮01⋮00)	(1111⋮0110)
(00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮11⋮00)	(1110⋮0111)
(00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮10)	(0001⋮1100)
(00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮01)	(1111⋮0011)
(00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮00⋮11)	(1110⋮1111)

Note. Single i-spotty-byte errors considered in Example 3.1 can also be corrected by using double error correcting BCH code of length 90. But for a BCH code of length $N = 90 \leq 2^7 - 1$, we require atmost $2 \times 7 = 14$ parity check digits while here i-spotty-byte code of the same length $N = 90$ requires only 8 check bits.

Conclusion. In this paper, we have discussed the code construction method of i-spotty-byte error correcting codes in terms of their parity check matrix. The method has also been illustrated with the help of an example.

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